


# An Economic Derivation on Trade Coefficients under the Framework of Multi-regional I-O Analysis

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DISCUSSION PAPER No. 29

**An Economic Derivation of Trade Coefficients under the Framework of Multi-regional I-O Analysis<sup>\*</sup>**

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**Abstract**

The gravity model, entropy model, potential type model and others like these have been adopted to formulate interregional trade coefficients under the framework of Multi-Regional I-O (MRIO) analysis. Since most of these models are based upon analogies in physics or on statistical principles, they do not provide a theoretical explanation from the view of a firm's or individual's rational and deterministic decision making. In this paper, according to the deterministic choice theory, not only is an alternative formulation of the trade coefficients presented, but also a discussion of an appropriate definition for purchasing prices indices. Since this formulation is consistent with the MRIO system, it can be employed as a useful model-building tool in multi-regional models such as the spatial CGE model.

**Keywords:** trade coefficients, multi-regional, input-output, Armington assumption

**JEL classification:** C67, C68

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# 1 Introduction

As a universal social phenomenon, the spatial interaction of persons and things such as population migration, the flow of goods, money, information, traffic movement and tourist travel, have been treated as important themes. They have been studied by economists, demographers, geographers, sociologists and others. In order to describe such interactions from the view of behavioral science, many models have been developed.

The earliest statement of human interaction seems to have been made by Carey in 1858(see Niedercorn and Bechdolt[1]). He defined the “gravity law” of spatial interaction which was originally derived from and analogous to Newton’s law of the gravitational force  $F_{ij}$  between two masses  $m_i$  and  $m_j$  separated by a distance  $d_{ij}$ . It can be written as the following simple form, where  $r$  is a constant.

$$F_{ij} = \gamma \frac{m_i m_j}{d_{ij}^2} \quad (1)$$

Carey believed that man is to society as a molecule is to matter. The more persons concentrated into a given area, the more attractive force exerted by that area. Equation (1) can be interpreted to mean that the degree of attractive force ( $F$ ) varies directly with the concentration of persons ( $m$ ), and inversely with distance ( $d$ ).

Later writers developed, expanded, modified, and applied these concepts. These writers include Young[2], Zipf[3], Anderson[4], Harris[5], Isard[6] and others. However, most of their work is based upon analogies in physics laws or upon statistics principles. They suffer from a lack of firm theoretical foundation, especially in the sense that they do not provide a theoretical explanation from the view of an individual’s rational decision making.

As a constructive experiment, Niedercorn and Bechdolt[1][7], Golob et al.[8] made some attempts to derive the “gravity law” of spatial interaction under the theoretical framework of deterministic utility theory. Later, many contributions were made by Smith[9], Isard[10], McFadden[11],[12], and others who expanded this approach into a new area by adopting probabilistic utility theory.

All of work mentioned above basically focuses on human travel or shopping behavior. A considerable number of studies have been conducted on the flow of goods, and this is the concern of this paper. As space is limited, focus will be placed on the formulation of trade coefficients under the framework of the Multi-Regional I-O (MRIO) analysis. This will be done by introducing and discussing representative researches.

Leontief and Strout[13] used the following general equation to formulate interregional trade flows.

$$T_i^{rs} = Q_i^{rs} \frac{X_i^r D_i^s}{\sum_r X_i^r}, \quad (r \neq s) \quad (2)$$

where the  $X_i^r$  represents the supply pool (see Batten and Boyce[15]) of goods  $i$  in region  $r$ ,  $D_i^s$  the demand pool of goods  $i$  in region  $s$ , and  $T_i^{rs}$  the total shipments of goods  $i$  from the supply pool in region  $r$  to the demand pool in region  $s$ . The economy is closed, so  $\sum_r X_i^r = \sum_s D_i^s$ . The coefficients  $Q_i^{rs}$  are distance decay parameters which can be viewed as empirical constants and are negatively related to per-unit transportation costs. The above formulation can be considered a special instance of the gravity model. As a practical application, Okamoto[14] referred to Leontief and Strout's formulation in discussion of the non-survey estimation methodology used in the interregional I-O table for China.

However, several things need to be considered when introducing Leontief and Strout's gravity model to the MRIO system.

Following Moses[16], assume that each industry in region  $s$  consumes the same fraction of the import of goods  $i$  from region  $r$  so that the trade coefficients can be stated as follows (regardless of the final users):

$$t_{ij}^{rs} = \frac{T_{ij}^{rs}}{\sum_r T_{ij}^{rs}} = t_i^{rs} = \frac{T_i^{rs}}{\sum_r T_i^{rs}}. \quad (3)$$

Then substituting equation (2) into (3)

$$t_i^{rs} = \frac{\frac{X_i^r D_i^s}{\sum_r X_i^r} Q_i^{rs}}{\sum_{r \neq s} \frac{X_i^r D_i^s}{\sum_r X_i^r} Q_i^{rs} + T_i^{ss}}. \quad (4)$$

Combining  $t_i^{rs} D_i^s = T_i^{rs}$ , the above equation yields:

$$D_i^s = \sum_{r \neq s} \frac{X_i^r D_i^s}{\sum_r X_i^r} Q_i^{rs} + T_i^{ss}. \quad (5)$$

If  $X_i^r$ ,  $D_i^s$ ,  $T_i^{ss}$  and  $Q_i^{rs}$  are available, it is not difficult to obtain  $t_i^{rs}$  by equation (4). However the systematic statistical information on  $Q_i^{rs}$  is unavailable in reality. Many indirect attempts have been made to overcome this difficulty. For example, interregional distances, or more generally unit transport costs and others, can be used as proxy variables for  $Q_i^{rs}$ . Still a consistency problem occurs in this case. Whenever the estimated  $Q_i^{rs}$  have to satisfy equation (5),<sup>1</sup> the estimation of  $Q_i^{rs}$  become more difficult.

Differentiating the gravity model, Wilson[17] took another approach called the entropy-maximizing model for projecting interregional trade flows. The most general form of this model is as follows.

$$\text{maximize } E = - \sum_i \sum_r \sum_s T_i^{rs} \ln T_i^{rs} \quad (6)$$

$$\text{subject to : } \sum_s T_i^{sr} = \sum_j a_{ij}^r \sum_s T_i^{rs} + y_i^r \quad (7)$$

$$\sum_r \sum_s T_i^{rs} c_i^{rs} = C_i \quad (8)$$

where  $E$  is entropy,  $T_i^{rs}$  the shipments of goods  $i$  from region  $r$  to  $s$ ,  $a_{ij}^r$  the input coefficients in region  $s$ ,  $y_i^r$  the final demand for goods  $i$  in region  $r$ ,  $c_i^{rs}$  the costs required to transport one unit of goods  $i$  from  $r$  to  $s$ ,  $C_i$  the total transport costs for goods  $i$  given from outside of the model. As solutions to the above, interregional trade flows are written as follows:

$$T_i^{rs} = \lambda_i^r \mu_i^s \exp(-\eta_i c_i^{rs}), \quad (\lambda_i^r, \eta_i > 0) \quad (9)$$

where  $\lambda_i^r$  and  $\eta_i$  are Lagrange multipliers associated with equations (7) and (8) respectively. Based on this model, many studies related to the projection

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<sup>1</sup>According to the gravity law, it can be imaged that the force between an object and itself is  $\infty$ . In this case, the distance  $d_{ij}$  between the entities equals 0. From this viewpoint, the gravity model is difficult to use for representation of intra-regional trade flows.

of interregional trade flows have been made. These include Sasaki, et al.[18], Okuda[19] and others(just to name a few). Substituting equation (9) into (3), produces

$$t_i^{rs} = \frac{\lambda_i^r \exp(-\eta_i c_i^{rs})}{\sum_r \lambda_i^r \exp(-\eta_i c_i^{rs})} \quad (10)$$

which implies that trade coefficients depend on transport costs only.

By assuming that trade coefficients are negatively correlated with purchasing prices  $p_i^r + c_i^{rs}$  of the commodities produced in respective regions and positively correlated with the production capacity  $X_i^r$  (which can be regarded as the specific potential of region  $r$ ), Amano and Fujita[20] proposed the following for formulating trade coefficients:

$$t_i^{rs} = \kappa_i^s X_i^r \exp(-\zeta_i(p_i^r + c_i^{rs})), \quad (\zeta_i > 0). \quad (11)$$

From the condition  $\sum_r t_i^{rs} = 1$ ,  $t_i^{rs}$  can be shown as follows:

$$t_i^{rs} = \frac{X_i^r \exp(-\zeta_i(p_i^r + c_i^{rs}))}{\sum_r X_i^r \exp(-\zeta_i(p_i^r + c_i^{rs}))} \quad (12)$$

where  $\kappa_i^s$ ,  $\zeta_i$  are parameters. Such a potential model is well known and widely used in projecting interregional trade flows. For example, Ando and Shibata[21], [22], Mizokami[23], Meng and Ando[24] and others have used this model.

The formulations of trade coefficients found above can be classified according to statistical concept or scientific field relative to the framework of MRIO analysis.

As Figure 1 indicates, both the gravity model and the entropy model are analogous to physical relationships from the view of social physics. However the former is deterministic, while the latter is probabilistic (statistical).

Few studies have used the multinomial logit model (according to McFadden [11], [12]) to formulate trade coefficients based on probabilistic choice theory, though theoretically this is possible. Considering that the logit model is a probabilistic approach, it is difficult to maintain consistency with MRIO analysis which is generally considered to be deterministic. This paper offers an alternative formulation of trade coefficients according to deterministic choice theory completely within the framework of MRIO analysis.

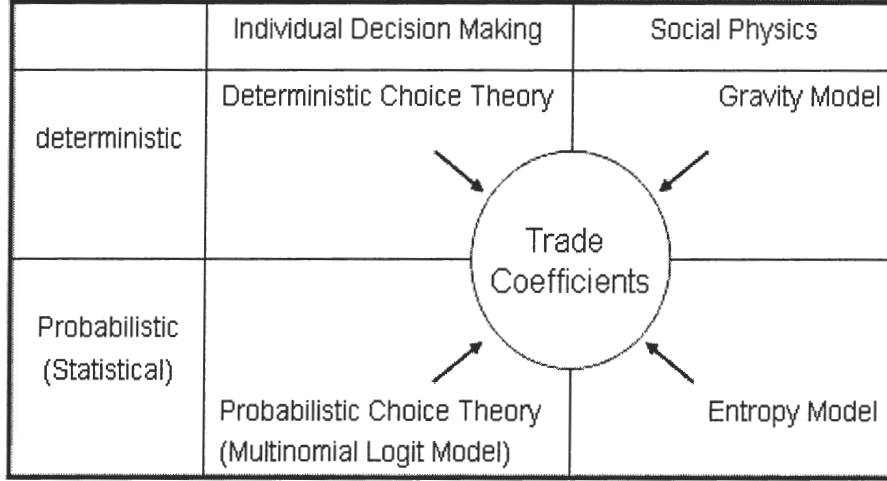


Figure 1: classification

The remainder of this paper is organized as follows: Section 2 describes some important assumptions used in formulation. Section 3 proposes a simple profit maximization model to formulate trade coefficients, and at the same time derive the relationship between purchasing prices and production prices by the cost minimization principle. Section 4 presents conclusions.

## 2 Assumptions

There are two important assumptions in MRIO analysis: the so-called Chenery-Moses Assumption, and the Armington Assumption[25]. These are used to formulate the trade coefficients in the next section, so a discussion of their properties is included here.

### 2.1 The Chenery-Moses Assumption

Since complete details of interdependences among sectors and regions is not generally attainable, it is customary to introduce the so-called Chenery-Moses Assumption into MRIO analysis. This assumption defines intermediate and final demand through a set of trade coefficients, each expressing the share of



demand in a region satisfied by production in another region. This is represented as follows:

$$x_{ij}^{rs} = a_{ij}^{rs} X_j^s = t_i^{rs} a_{ij}^s X_j^s \quad (13)$$

where  $x_{ij}^{rs}$  is the input of goods  $i$  in sector  $j$  of region  $s$  from region  $r$ ,  $a_{ij}^{rs}$  the interregional input-output coefficients, and  $a_{ij}^s$  the input-output coefficients in region  $r$ .

Under the MRIO system, the above equation shows the relationship between so-called non-competitive import type and competitive import types. It can be considered to convert between the Isard type I-O table and the Chenery-Moses type I-O table.

A peculiar theoretical problem of MRIO analysis stems from the simple fact that identical goods can be, and actually are, produced and consumed in different regions. In a competitive import scheme, goods are considered to be “perfectly” homogeneous, so they can be “perfectly” substituted for each other. Given this, If regions are defined as locational points, and all shipments are assumed to result from strictly rational decisions based on perfect information, then cross shipments can not occur.

In actual empirical analysis, especially within the framework of the MRIO system, good  $i$  from different regions will generally be defined as an aggregate of several similar but not strictly identical items. That is because in reality, the classification of goods (sectors) is rough, and also because regions  $r$  and  $s$  will often represent more or less extended areas. Thus the average distance (or the average unit costs of transportation) between regions would necessarily conceal the actual diversity of commodity flows connecting a great many distinct pairs of sending and receiving points. Further the MRIO table is a record of interregional and inter-industrial transactions over a one-year period, a time lag exists in various regions for the supply of goods. Under such circumstances, cross shipments should be expected (and are actually observed), nearly everywhere.

## 2.2 The Armington Assumption

For decades, trade economists have modeled imperfect substitutions among different regions and called it the Armington Assumption. Many national CGE models have employed this assumption for modeling foreign trade. Expanding this idea, Miyagi and Honbu[26], Bröcker[27] and others have attempted to formulate trade coefficients by a cost minimization of transport firms subject to CES production technology in their spatial CGE models. However the transport firms are considered an imaginary agent that does not use any resource to produce transport services. Thus it is difficult to be used in realistic I-O analysis. since the production structure of the transportation sector has not been taken into account explicitly. In addition, their results in terms of trade coefficients do not support the condition  $\sum_r t_i^{rs} = 1$  directly.

## 3 The simple model

In this section, the Armington Assumption is introduced into a firm's decision making in order to formulate interregional trade coefficients more realistically and make them consistent with the MRIO system.

### 3.1 Approach by profit maximization

Consider an economy with  $m$  regions,  $r(s) = 1, \dots, m$ , and  $n$  industries (goods<sup>2</sup>),  $j(i) = 1, \dots, n$ . Each output in each region is assumed to be produced according to the following production function:

$$X_j^s = \prod_i \left( \sum_r (\nu_{ij}^{rs})^{1+\rho_{ij}^s} (x_{ij}^{rs})^{-\rho_{ij}^s} \right)^{-\frac{\alpha_{ij}^s}{\rho_{ij}^s}} (K_j^s)^{\alpha_{Kj}^s} (L_j^s)^{\alpha_{Lj}^s} \quad (14)$$

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<sup>2</sup>Here,  $i$  excludes the transport sector. In considering the role of the transport sector and f.o.b./c.i.f. price differentials explicitly, we assume that all demand to transport sector is derived from demand for other commodities, and the prices that suppliers and demanders face differ as much as the fare associated with commodities.

where  $X_j^s$  denotes the amount of output produced by industry  $j$  located in region  $r$ ,  $x_{ij}^{rs}$  the intermediate purchase of the output  $i$ <sup>3</sup> by the industry  $j$  located in region  $s$ , the output  $i$  is produced in region  $r$  and shipped to region  $s$ .  $K_j^s$ ,  $L_j^s$  are respectively the capital input and labor input employed by the industry  $j$  in region  $s$ .

The production function includes both the lower-level and the upper-level. The upper-level is a Cobb-Douglas type technology, and the low-level for intermediate inputs employs a CES type technology. Here,  $\nu_{ij}^{rs}$  represent the share parameters and  $\rho_{ij}^s$  the substitution parameters satisfying the following conditions respectively,

$$\nu_{ij}^{rs} > 0, \quad -1 \leq \rho_{ij}^s < \infty. \quad (15)$$

( $\rho_{ij}^s = -1$ ) represents the case for perfect substitutes of the intermediate inputs. Linear homogeneity on the production frontier is supposed:

$$\sum_i \alpha_{ij}^s + \alpha_{Kj}^s + \alpha_{Lj}^s = 1. \quad (16)$$

For simplicity, assume that all production factors are immobile across regions and industries, and that the parameters  $\rho_{ij}^s = \rho_i$  and  $\nu_{ij}^{rs} = \nu_i^r$ . The two assumptions do not affect results and can be relaxed.

Define  $p_j^s$ ,  $\gamma_j^s$  and  $\omega_j^s$  to be prices of  $X_j^s$ ,  $K_j^s$  and  $L_j^s$  respectively. Given producer price  $p_j^s$ , purchasing price  $p_i^r + c_i^{rs}$ , capital rent  $\gamma_j^s$  and wage rate  $\omega_j^s$ , firms are assumed to choose the profit maximizing level of output  $X_j^s$ , intermediate purchase  $x_{ij}^{rs}$ , labor  $L_j^s$  and capital  $K_j^s$ . This becomes a profit maximization model with technical constraints (14) and (16), i.e.,

$$\pi_j^s = p_j^s X_j^s - \sum_i \sum_r (p_i^r + c_i^{rs}) x_{ij}^{rs} - \gamma_j^s K_j^s - \omega_j^s L_j^s. \quad (17)$$

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<sup>3</sup>The construction sector ( $i = Con.$ ) requires some specific handling in the MIOR system. In input-output tables, for many countries (like Japanese), measure the outputs of this sector at the sites of construction. Accordingly, trade of its outputs is non-existent by definition. Moreover, those outputs will never be used as the inputs to other industrial activities, and all of them go to capital formations. Thus the sectors for which the product in Eq.(14) is taken may be written as  $i \neq Con.$ . However, construction firms themselves behave exactly the same way as other firms.

The first-order condition for profit maximizing yields

$$\frac{\partial \pi_j^s}{\partial x_{ij}^{rs}} = p_j^s \alpha_{ij}^s \frac{X_j^s}{x_{ij}^{rs}} \frac{(\nu_i^r)^{1+\rho_i} (x_{ij}^{rs})^{-\rho_i}}{\sum_r (\nu_i^r)^{1+\rho_i} (x_{ij}^{rs})^{-\rho_i}} - (p_i^r + c_i^{rs}) = 0. \quad (18)$$

Then from equation (13) and (18),

$$\alpha_{ij}^s = \frac{(p_i^r + c_i^{rs})(t_i^{rs})^{1+\rho_i} \sum_r (\nu_i^r)^{1+\rho_i} (t_i^{rs})^{-\rho_i}}{(\nu_i^r)^{1+\rho_i} \cdot p_j^s} a_{ij}^s. \quad (19)$$

The above Lagrangian solution for  $x_{ij}^{rs}$  is available to any region  $r'$ . Thus a similar result for  $x_{ij}^{r's}$  (refer to (19)) is obtained:

$$\alpha_{ij}^s = \frac{(p_i^{r'} + c_i^{r's})(t_i^{r's})^{1+\rho_i} \sum_r (\nu_i^r)^{1+\rho_i} (t_i^{rs})^{-\rho_i}}{(\nu_i^{r'})^{1+\rho_i} \cdot p_j^s} a_{ij}^s. \quad (20)$$

Dividing (20) by (19),

$$\frac{t_i^{r's}}{t_i^{rs}} = \frac{\nu_i^{r'}}{\nu_i^r} \left( \frac{p_i^r + c_i^{rs}}{p_i^{r'} + c_i^{r's}} \right)^{\frac{1}{1+\rho_i}}. \quad (21)$$

Using the condition  $\sum_r t_i^{r's} = 1$ , arranging and transforming the above equation, trade coefficients are obtained as follows:

$$t_i^{rs} = \frac{\nu_i^r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}{\sum_r \nu_i^r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}. \quad (22)$$

The above equation indicates that the trade coefficients depend on production prices  $p_i^r$  and transport costs  $c_i^{rs}$ . In the special case of perfect substitution for interregional inputs:  $\rho_i = -1$ ,  $\nu_i^r = 1; \forall r$ , the formulation of trade coefficients (22) is estimated as follows:

$$t_i^{rs} = \begin{cases} 0 & \text{for } (p_i^r + c_i^{rs}) > \min_{r \in R} (p_i^r + c_i^{rs}) \\ \frac{1}{\#\{r \mid (p_i^r + c_i^{rs}) = \min_{r \in R} (p_i^r + c_i^{rs})\}} & \text{for } (p_i^r + c_i^{rs}) = \min_{r \in R} (p_i^r + c_i^{rs}) \end{cases} \quad (23)$$

where,  $\#r$  denotes the number of regions. The above equation implies that under the perfect substitution, if the purchasing price  $p_i^r + c_i^{rs}$  in region  $s$  for goods  $i$  produced in region  $r$  is bigger than the smallest purchasing price of goods  $i$  produced among all the regions, then there no trade-flow from region  $r$  to  $s$  exists. On the other hand, if the purchasing price  $p_i^r + c_i^{rs}$  is the smallest

one and the only one, then region  $s$  will import all needed goods from region  $r$  alone. Further, if the number of regions who have the same smallest purchasing prices is  $r$ , then region  $s$  will import  $1/r$  from each related region.

Alternatively, the total of intermediate inputs in the production function (14) can be considered as a aggregate  $D_{ij}^s$ , as

$$D_{ij}^s = \left( \sum_r (\nu_i^r)^{1+\rho_i} (x_{ij}^{rs})^{-\rho_i} \right)^{-\frac{1}{\rho_i}}. \quad (24)$$

The profit function (17) may be rewritten as follows:

$$\pi_j^s = p_j^s X_j^s - \sum_{i \neq 5} q_i^s D_{ij}^s - \gamma_j^s K_j^s - \omega_j^s L_j^s \quad (25)$$

where  $q_i^s$  denotes purchasing prices of aggregate goods. It is consider to be the purchasing price index of the aggregate good  $D_{ij}^s$  in region  $s$ . According to the first-order conditions for a maximum,

$$\alpha_{ij}^s = \frac{q_i^s D_{ij}^s}{p_j^s X_j^s}, \quad \alpha_{Kj}^s = \frac{\gamma_j^s K_j^s}{p_j^s X_j^s}, \quad \text{and} \quad \alpha_{Lj}^s = \frac{\omega_j^s L_j^s}{p_j^s X_j^s}. \quad (26)$$

The left sides of the above equations are parameters in the production function. According to the right sides, they can be regarded as monetary input-output coefficients. Note that, the physical input-output coefficients are defined as follows:

$$a_{ij}^s = \frac{D_{ij}^s}{X_j^s}, \quad a_{Kj}^s = \frac{K_j^s}{X_j^s}, \quad \text{and} \quad a_{Lj}^s = \frac{L_j^s}{X_j^s}. \quad (27)$$

The relations between the monetary and the physical input-output coefficients may be written, respectively, in the following forms:

$$a_{ij}^s = \frac{p_j^s}{q_i^s} \alpha_{ij}^s, \quad a_{Kj}^s = \frac{p_j^s}{\gamma_j^s} \alpha_{Kj}^s, \quad \text{and} \quad a_{Lj}^s = \frac{p_j^s}{\omega_j^s} \alpha_{Lj}^s. \quad (28)$$

### 3.2 Approach by cost minimization

It is well known in modern microeconomics that a duality exists between production and cost functions. According to what is known as Shephard's duality, the unit cost function can be represented as follows:

$$p_j^s = \prod_i \left[ \frac{1}{\alpha_{ij}} \left( \sum_r \nu_i^r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}} \right)^{\frac{1+\rho_i}{\rho_i}} \right]^{\alpha_{ij}} \left[ \frac{\gamma_j^s}{\alpha_{Kj}^s} \right]^{\alpha_{Kj}^s} \left[ \frac{\omega_j^s}{\alpha_{Lj}^s} \right]^{\alpha_{Lj}^s}. \quad (29)$$

Shephard's lemma may also be employed to obtain the unit demand function for input  $x_{ij}^{rs}$  shown below. This theoretically equals the interregional input-output coefficient:

$$\frac{\partial p_j^s}{\partial(p_i^r + c_i^{rs})} = \frac{\alpha_{ij} p_j^s}{p_i^r + c_i^{rs}} \frac{\nu_i^r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}{\sum_r \nu_i^r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}} = a_{ij}^{rs} . \quad (30)$$

Using equations (3) and (28) to arrange the above equation,

$$t_i^{rs} = \frac{q_i^s}{p_i^r + c_i^{rs}} \frac{\nu_i^r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}{\sum_r \nu_i^r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}} . \quad (31)$$

Moving the term  $p_i^r + c_i^{rs}$  to the left side and computing  $\sum_r$  for both sides,

$$q_i^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs} . \quad (32)$$

This implies that purchasing price indices can be considered as an average value of the purchasing prices weighted by the trade coefficients. Since  $\sum_r t_i^{rs} = 1$ , then both sides may be directly summarize(31) by  $r$ , and a different expression of the purchasing prices indices is as follows:

$$q_i^s = \frac{\sum_r \nu_i^r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}{\sum_r \nu_i^r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}} . \quad (33)$$

Further substituting the above equation into equation (31) to calculate  $t_i^{rs}$  results in the following:

$$t_i^{rs} = \frac{\nu_i^r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}{\sum_r \nu_i^r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}} .$$

This is the same as earlier equation (22).

The above formulation of trade coefficients can also be obtained by solving the household utility maximization problem. As an extension, the behaviors of the transport sector, government, investor, foreign economy, and other may be considered in order to build a spatial general equilibrium model (see Meng and Ando[28]).

## 4 Conclusion

Though only a simple model has been presented here, it should be clear that interregional trade coefficients can be logically derived from the economic principle of firms' (individuals') deterministic decision making under the framework of MRIO analysis, rather than from the vague and irrelevant concepts of social physics.

What is particularly nice about these results is that the formulation of trade coefficients is simple and useful. Unlike the logit or entropy model, the formulation presented here, does not include any probabilistic form. It depends on production prices and transport costs only. As a model-building tool, it can be easily employed for dealing with interregional trade-flow easily in spatial CGE models.

An expression of purchasing price indices was also derived in this paper. Purchasing price indices of aggregate goods can be considered as an average of production prices including transport costs weighted by trade coefficients. This result provides a new idea for describing the spatial price equilibrium within the MRIO system.

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